Uncertainty in satellite estimate of Global Mean Sea Level changes, trend and acceleration

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Overview

- GMSL from T/P, Jason-1, 2 & 3,
- Solutions from different groups are not independent,
- Can we estimate reliable uncertainties for GMSL timeseries?
- essential for budget closure, detection & attribution, ...
Mathematical elements

- Express SLA as forced response, plus IV, plus error:
  \[ Y = AX + IV + E \]
- Then the forced response is given by:
  \[ \hat{A} = (X^tX)^{-1}X^tY \]
- And its variance-covariance by:
  \[ \Omega_{\hat{A}} = (X^tX)^{-1}X^tC_EX(X^tX)^{-1} \]
Mathematical elements

\[ \Omega_{\hat{A}} = (X^t X)^{-1} X^t C_E X (X^t X)^{-1} \]

- \( C_E \) is the variance-covariance matrix of errors,

- Goal: estimate \( C_E \)
- Method: combine elementary error terms
- Measurement system errors only, no internal variability
## Altimetry Errors

<table>
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<tr>
<th>Error source</th>
<th>Category</th>
<th>Magnitude (at 1 $\sigma$)</th>
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</table>
| High frequency errors: altimeter noise, geophysical corrections, orbits ...  | correlated error ($\lambda = 2$ months) | $\sigma = 1.7$ mm for TOPEX period  
$\sigma = 1.5$ mm for Jason-1 period  
$\sigma = 1.2$ mm for Jason-2/3 period | Cal/Val activities                             |
| Medium frequency errors: geophysical corrections, orbits ...                 | correlated error ($\lambda = 1$ year) | $\sigma = 1.3$ mm for TOPEX period  
$\sigma = 1.2$ mm for Jason-1 period  
$\sigma = 1$ mm for Jason-2/3 period | Cal/val activities                              |
| Large frequency errors: wet troposphere correction                          | correlated error ($\lambda = 5$ years) | $\sigma = 1.1$ mm over all the period (\(\Rightarrow\) to 0.2 mm/yr for 10 years)         | Legeais et al., 2015, Thao et al., 2014         |
| Large frequency errors: orbits (gravity fields)                             | correlated error ($\lambda = 10$ years) | $\sigma = 1.12$ mm over TOPEX period (no GRACE data)  
$\sigma = 0.5$ mm over Jason period (\(\Rightarrow\) to 0.05 mm/yr for 10 years) | Couhert al., 2015  
Rudenko et al., 2017                                                      |
| Altimeter instabilities on TOPEX-A/B                                         | drift error                     | $\sigma = 0.7$ mm/yr on TOPEX-A period (*)  
$\sigma = 0.1$ mm/yr on TOPEX-B period | Watson et al., 2014 ; Beckley et al., 2017; Ablain et al. 2017 |
| Long-term drift errors: orbit (ITRF and GIA)                                 | drift error                     | $\sigma = 0.12$ mm/yr over all the period                                      | Couhert et al., 2015  
Spada, 2017                                                                 |
| Error when linking altimetric missions together.                             | bias errors                     | $\sigma = 2$ mm for TP-A/TP-B  
$\sigma = 0.5$ mm for TP-B/J1, J1/J2, J2/J3.                                      | Zawadzki et al., 2018                           |
Error Covariance

- Total error covariance,
- Sum of all individual contributions,
- Is used to estimate 90% CI on trends and acceleration,
- Errors covary from one end of the record to the other,
- Available to users
  https://doi.org/10.17882/58344
Error envelope

- Error envelope as the square root of matrix diagonal,
- Corrections for the TOPEX-A drift fall outside the enveloppe
GMSL trend uncertainty

- 25-yr record: \(3.35 \pm 0.4 \text{ mm/yr}\)
- Uncertainties decrease over time
- [1998-2018] is the most accurate \(\pm 0.35 \text{ mm/yr}\)

Halfwidth of 90% conf. int.
GMSL acceleration uncertainty

Acceleration uncertainty on periods of 10 years and longer
When are accelerations detected?

- Any 10+ year record including 2018 shows significant acceleration
- Recovery from Pinatubo is detected
- Significant acceleration on the 25-yr record
  \[ 0.12 \pm 0.07 \text{ mm/yr}^2 \]
Conclusions

• Error covariance description, available to users
  – To derive confidence levels on any metric
  – [https://doi.org/10.17882/58344](https://doi.org/10.17882/58344)

• Used to derive confidence on GMSL trend and acceleration
  – $3.35 \pm 0.4$ mm/yr, $0.12 \pm 0.07$ mm/yr$^2$
  – On 25 years, at the 90% CL

• Uncertainty built from current knowledge of system errors,
  – should be revised and updated to reflect new findings,
  – does not include internal variability,
  – there are uncertainties on the error budget
I don't know how to propagate error correctly, so I just put error bars on all my error bars.

https://xkcd.com/2110/